
factor_analyzer Documentation

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This is a Python module to perform exploratory and factor analysis (EFA), with several optional rotations. It also includes a class to perform confirmatory factor analysis (CFA), with certain pre-defined constraints. In exploratory factor analysis, factor extraction can be performed using a variety of estimation techniques. The `factor_analyzer` package allows users to perform EFA using either (1) a minimum residual (MINRES) solution, (2) a maximum likelihood (ML) solution, or (3) a principal factor solution. However, CFA can only be performed using an ML solution.

Both the EFA and CFA classes within this package are fully compatible with *scikit-learn*. Portions of this code are ported from the excellent R library *psych*, and the *sem* package provided inspiration for the CFA class.

Exploratory factor analysis (EFA) is a statistical technique used to identify latent relationships among sets of observed variables in a dataset. In particular, EFA seeks to model a large set of observed variables as linear combinations of some smaller set of unobserved, latent factors. The matrix of weights, or factor loadings, generated from an EFA model describes the underlying relationships between each variable and the latent factors.

Confirmatory factor analysis (CFA), a closely associated technique, is used to test an a priori hypothesis about latent relationships among sets of observed variables. In CFA, the researcher specifies the expected pattern of factor loadings (and possibly other constraints), and fits a model according to this specification.

Typically, a number of factors (K) in an EFA or CFA model is selected such that it is substantially smaller than the number of variables. The factor analysis model can be estimated using a variety of standard estimation methods, including but not limited to MINRES or ML.

Factor loadings are similar to standardized regression coefficients, and variables with higher loadings on a particular factor can be interpreted as explaining a larger proportion of the variation in that factor. In the case of EFA, factor loading matrices are usually rotated after the factor analysis model is estimated in order to produce a simpler, more interpretable structure to identify which variables are loading on a particular factor.

Two common types of rotations are:

1. The **varimax** rotation, which rotates the factor loading matrix so as to maximize the sum of the variance of squared loadings, while preserving the orthogonality of the loading matrix.
2. The **promax** rotation, a method for oblique rotation, which builds upon the varimax rotation, but ultimately allows factors to become correlated.

This package includes a `factor_analyzer` module with a stand-alone `FactorAnalyzer` class. The class includes `fit()` and `transform()` methods that enable users to perform factor analysis and score new data using the fitted factor model. Users can also perform optional rotations on a factor loading matrix using the `Rotator` class.

The following rotation options are available in both `FactorAnalyzer` and `Rotator`:

- (a) `varimax` (orthogonal rotation)
- (b) `promax` (oblique rotation)
- (c) `oblimin` (oblique rotation)

- (d) oblimax (orthogonal rotation)
- (e) quartimin (oblique rotation)
- (f) quartimax (orthogonal rotation)
- (g) equamax (orthogonal rotation)
- (h) geomin_obl (oblique rotation)
- (i) geomin_ort (orthogonal rotation)

In addition, the package includes a `confirmatory_factor_analyzer` module with a stand-alone `ConfirmatoryFactorAnalyzer` class. The class includes `fit()` and `transform()` that enable users to perform confirmatory factor analysis and score new data using the fitted model. Performing CFA requires users to specify in advance a model specification with the expected factor loading relationships. This can be done using the `ModelSpecificationParser` class.

CHAPTER 2

Requirements

- Python 3.4 or higher
- numpy
- pandas
- scipy
- scikit-learn

You can install this package via pip with:

```
$ pip install factor_analyzer
```

Alternatively, you can install via conda with:

```
$ conda install -c ets factor_analyzer
```

3.1 factor_analyzer package

3.1.1 factor_analyzer.analyze Module

3.1.2 factor_analyzer.factor_analyzer Module

Factor analysis using MINRES or ML, with optional rotation using Varimax or Promax.

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organization Educational Testing Service

date 2021-10-18

```
class factor_analyzer.factor_analyzer.FactorAnalyzer (n_factors=3,          ro-
                                                    rotation='promax',
                                                    method='minres',
                                                    use_smc=True,
                                                    is_corr_matrix=False,
                                                    bounds=(0.005,          1),
                                                    impute='median',
                                                    svd_method='randomized',
                                                    rotation_kwargs=None)
Bases: sklearn.base.BaseEstimator, sklearn.base.TransformerMixin
```

A FactorAnalyzer class, which -

- (1) Fits a factor analysis model using minres, maximum likelihood, or principal factor extraction and returns the loading matrix
- (2) Optionally performs a rotation, with method including:
 - (a) varimax (orthogonal rotation)
 - (b) promax (oblique rotation)
 - (c) oblimin (oblique rotation)
 - (d) oblimax (orthogonal rotation)
 - (e) quartimin (oblique rotation)
 - (f) quartimax (orthogonal rotation)
 - (g) equamax (orthogonal rotation)

Parameters

- **n_factors** (*int*, *optional*) – The number of factors to select. Defaults to 3.
- **rotation** (*str*, *optional*) – The type of rotation to perform after fitting the factor analysis model. If set to None, no rotation will be performed, nor will any associated Kaiser normalization.

Methods include:

- (a) varimax (orthogonal rotation)
- (b) promax (oblique rotation)
- (c) oblimin (oblique rotation)
- (d) oblimax (orthogonal rotation)
- (e) quartimin (oblique rotation)
- (f) quartimax (orthogonal rotation)
- (g) equamax (orthogonal rotation)

Defaults to ‘promax’.

- **method** (*{'minres', 'ml', 'principal'}*, *optional*) – The fitting method to use, either MINRES or Maximum Likelihood. Defaults to ‘minres’.
- **use_smc** (*bool*, *optional*) – Whether to use squared multiple correlation as starting guesses for factor analysis. Defaults to True.
- **bounds** (*tuple*, *optional*) – The lower and upper bounds on the variables for “L-BFGS-B” optimization. Defaults to (0.005, 1).
- **impute** (*{'drop', 'mean', 'median'}*, *optional*) – If missing values are present in the data, either use list-wise deletion (‘drop’) or impute the column median (‘median’) or column mean (‘mean’). Defaults to ‘median’
- **use_corr_matrix** (*bool*, *optional*) – Set to true if the *data* is the correlation matrix. Defaults to False.
- **svd_method** (*{'lapack', 'randomized'}*) – The SVD method to use when *method*=‘principal’. If ‘lapack’, use standard SVD from `scipy.linalg`. If ‘randomized’, use faster `randomized_svd` function from `scikit-learn`. The latter should only

be used if the number of columns is greater than or equal to the number of rows in the dataset. Defaults to 'randomized'

- **optional** (*rotation_kwargs*,) – Additional key word arguments are passed to the rotation method.

loadings

The factor loadings matrix. Default to None, if *fit()* has not been called.

Type numpy array

corr

The original correlation matrix. Default to None, if *fit()* has not been called.

Type numpy array

rotation_matrix

The rotation matrix, if a rotation has been performed.

Type numpy array

structure

The structure loading matrix. This only exists if the rotation is promax.

Type numpy array or None

psi

The factor correlations matrix. This only exists if the rotation is oblique.

Type numpy array or None

Notes

This code was partly derived from the excellent R package *psych*.

References

[1] <https://github.com/cran/psych/blob/master/R/fa.R>

Examples

```
>>> import pandas as pd
>>> from factor_analyzer import FactorAnalyzer
>>> df_features = pd.read_csv('tests/data/test02.csv')
>>> fa = FactorAnalyzer(rotation=None)
>>> fa.fit(df_features)
FactorAnalyzer(bounds=(0.005, 1), impute='median', is_corr_matrix=False,
               method='minres', n_factors=3, rotation=None, rotation_kwargs={},
               use_smc=True)
>>> fa.loadings_
array([[ -0.12991218,  0.16398154,  0.73823498],
       [ 0.03899558,  0.04658425,  0.01150343],
       [ 0.34874135,  0.61452341, -0.07255667],
       [ 0.45318006,  0.71926681, -0.07546472],
       [ 0.36688794,  0.44377343, -0.01737067],
       [ 0.74141382, -0.15008235,  0.29977512],
       [ 0.741675   , -0.16123009, -0.20744495],
```

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```

    [ 0.82910167, -0.20519428,  0.04930817],
    [ 0.76041819, -0.23768727, -0.1206858 ],
    [ 0.81533404, -0.12494695,  0.17639683]])
>>> fa.get_communalities()
array([0.588758 , 0.00382308, 0.50452402, 0.72841183, 0.33184336,
       0.66208428, 0.61911036, 0.73194557, 0.64929612, 0.71149718])

```

fit (*X*, *y=None*)

Fit the factor analysis model using either minres, ml, or principal solutions. By default, use SMC as starting guesses.

Parameters

- **x** (*array-like*) – The data to analyze.
- **y** (*ignored*) –

Examples

```

>>> import pandas as pd
>>> from factor_analyzer import FactorAnalyzer
>>> df_features = pd.read_csv('tests/data/test02.csv')
>>> fa = FactorAnalyzer(rotation=None)
>>> fa.fit(df_features)
FactorAnalyzer(bounds=(0.005, 1), impute='median', is_corr_matrix=False,
               method='minres', n_factors=3, rotation=None, rotation_kwargs={},
               use_smc=True)
>>> fa.loadings_
array([[ -0.12991218,  0.16398154,  0.73823498],
       [ 0.03899558,  0.04658425,  0.01150343],
       [ 0.34874135,  0.61452341, -0.07255667],
       [ 0.45318006,  0.71926681, -0.07546472],
       [ 0.36688794,  0.44377343, -0.01737067],
       [ 0.74141382, -0.15008235,  0.29977512],
       [ 0.741675 , -0.16123009, -0.20744495],
       [ 0.82910167, -0.20519428,  0.04930817],
       [ 0.76041819, -0.23768727, -0.1206858 ],
       [ 0.81533404, -0.12494695,  0.17639683]])

```

get_communalities ()

Calculate the communalities, given the factor loading matrix.

Returns communalities – The communalities from the factor loading matrix.

Return type numpy array

Examples

```

>>> import pandas as pd
>>> from factor_analyzer import FactorAnalyzer
>>> df_features = pd.read_csv('tests/data/test02.csv')
>>> fa = FactorAnalyzer(rotation=None)
>>> fa.fit(df_features)
FactorAnalyzer(bounds=(0.005, 1), impute='median', is_corr_matrix=False,
               method='minres', n_factors=3, rotation=None, rotation_kwargs={},

```

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```

        use_smc=True)
>>> fa.get_communalities()
array([0.588758 , 0.00382308, 0.50452402, 0.72841183, 0.33184336,
       0.66208428, 0.61911036, 0.73194557, 0.64929612, 0.71149718])

```

get_eigenvalues()

Calculate the eigenvalues, given the factor correlation matrix.

Returns

- **original_eigen_values** (*numpy array*) – The original eigen values
- **common_factor_eigen_values** (*numpy array*) – The common factor eigen values

Examples

```

>>> import pandas as pd
>>> from factor_analyzer import FactorAnalyzer
>>> df_features = pd.read_csv('tests/data/test02.csv')
>>> fa = FactorAnalyzer(rotation=None)
>>> fa.fit(df_features)
FactorAnalyzer(bounds=(0.005, 1), impute='median', is_corr_matrix=False,
               method='minres', n_factors=3, rotation=None, rotation_kwargs={},
               use_smc=True)
>>> fa.get_eigenvalues()
(array([ 3.51018854,  1.28371018,  0.73739507,  0.1334704 ,  0.03445558,
         0.0102918 , -0.00740013, -0.03694786, -0.05959139, -0.07428112]),
 array([ 3.51018905,  1.2837105 ,  0.73739508,  0.13347082,  0.03445601,
         0.01029184, -0.0074 , -0.03694834, -0.05959057, -0.07428059]))

```

get_factor_variance()

Calculate the factor variance information, including variance, proportional variance and cumulative variance for each factor

Returns

- **variance** (*numpy array*) – The factor variances.
- **proportional_variance** (*numpy array*) – The proportional factor variances.
- **cumulative_variances** (*numpy array*) – The cumulative factor variances.

Examples

```

>>> import pandas as pd
>>> from factor_analyzer import FactorAnalyzer
>>> df_features = pd.read_csv('tests/data/test02.csv')
>>> fa = FactorAnalyzer(rotation=None)
>>> fa.fit(df_features)
FactorAnalyzer(bounds=(0.005, 1), impute='median', is_corr_matrix=False,
               method='minres', n_factors=3, rotation=None, rotation_kwargs={},
               use_smc=True)
>>> # 1. Sum of squared loadings (variance)
... # 2. Proportional variance
... # 3. Cumulative variance
>>> fa.get_factor_variance()

```

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```
(array([3.51018854, 1.28371018, 0.73739507]),
array([0.35101885, 0.12837102, 0.07373951]),
array([0.35101885, 0.47938987, 0.55312938]))
```

get_uniquenesses()

Calculate the uniquenesses, given the factor loading matrix.

Returns uniquenesses – The uniquenesses from the factor loading matrix.

Return type numpy array

Examples

```
>>> import pandas as pd
>>> from factor_analyzer import FactorAnalyzer
>>> df_features = pd.read_csv('tests/data/test02.csv')
>>> fa = FactorAnalyzer(rotation=None)
>>> fa.fit(df_features)
FactorAnalyzer(bounds=(0.005, 1), impute='median', is_corr_matrix=False,
               method='minres', n_factors=3, rotation=None, rotation_kwargs={},
               use_smc=True)
>>> fa.get_uniquenesses()
array([0.411242, 0.99617692, 0.49547598, 0.27158817, 0.66815664,
       0.33791572, 0.38088964, 0.26805443, 0.35070388, 0.28850282])
```

transform(X)

Get the factor scores for new data set.

Parameters X (*array-like, shape (n_samples, n_features)*) – The data to score using the fitted factor model.

Returns X_new – The latent variables of X.

Return type numpy array, shape (n_samples, n_components)

Examples

```
>>> import pandas as pd
>>> from factor_analyzer import FactorAnalyzer
>>> df_features = pd.read_csv('tests/data/test02.csv')
>>> fa = FactorAnalyzer(rotation=None)
>>> fa.fit(df_features)
FactorAnalyzer(bounds=(0.005, 1), impute='median', is_corr_matrix=False,
               method='minres', n_factors=3, rotation=None, rotation_kwargs={},
               use_smc=True)
>>> fa.transform(df_features)
array([[ -1.05141425,  0.57687826,  0.1658788 ],
       [ -1.59940101,  0.89632125,  0.03824552],
       [ -1.21768164, -1.16319406,  0.57135189],
       ...,
       [  0.13601554,  0.03601086,  0.28813877],
       [  1.86904519, -0.3532394, -0.68170573],
       [  0.86133386,  0.18280695, -0.79170903]])
```

factor_analyzer.factor_analyzer.**calculate_bartlett_sphericity**(x)

Test the hypothesis that the correlation matrix is equal to the identity matrix.

H0: The matrix of population correlations is equal to I. H1: The matrix of population correlations is not equal to I.

The formula for Bartlett's Sphericity test is:

$$-1 * (n - 1 - ((2p + 5)/6)) * \ln(\det(R))$$

Where R det(R) is the determinant of the correlation matrix, and p is the number of variables.

Parameters *x* (*array-like*) – The array from which to calculate sphericity.

Returns

- **statistic** (*float*) – The chi-square value.
- **p_value** (*float*) – The associated p-value for the test.

`factor_analyzer.factor_analyzer.calculate_kmo(x)`

Calculate the Kaiser-Meyer-Olkin criterion for items and overall. This statistic represents the degree to which each observed variable is predicted, without error, by the other variables in the dataset. In general, a KMO < 0.6 is considered inadequate.

Parameters *x* (*array-like*) – The array from which to calculate KMOs.

Returns

- **kmo_per_variable** (*numpy array*) – The KMO score per item.
- **kmo_total** (*float*) – The KMO score overall.

3.1.3 factor_analyzer.confirmatory_factor_analyzer Module

Confirmatory factor analysis using machine learning methods.

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date 2021-10-18

class `factor_analyzer.confirmatory_factor_analyzer.ConfirmatoryFactorAnalyzer` (*specification=None, n_obs=None, is_cov_matrix=False, bounds=None, max_iter=200, tol=None, impute='median', disp=True*)

Bases: `sklearn.base.BaseEstimator`, `sklearn.base.TransformerMixin`

A `ConfirmatoryFactorAnalyzer` class, which fits a confirmatory factor analysis model using maximum likelihood.

Parameters

- **specification** (*ModelSpecification object or None, optional*) – A model specification. This must be a `ModelSpecification` object or `None`. If `None`, the `ModelSpecification` will be generated assuming that `n_factors == n_variables`, and that all variables load on all factors. Note that this could mean the factor model is not identified, and the optimization could fail. Defaults to `None`.

- **n_obs** (*int* or *None*, *optional*) – The number of observations in the original data set. If this is not passed and *is_cov_matrix=True*, then an error will be raised. Defaults to *None*.
- **is_cov_matrix** (*bool*, *optional*) – Whether the input *X* is a covariance matrix. If *False*, assume it is the full data set. Defaults to *False*.
- **bounds** (*list of tuples* or *None*, *optional*) – A list of minimum and maximum boundaries for each element of the input array. This must equal *x0*, which is the input array from your parsed and combined model specification. The length is:
$$((n_factors * n_variables) + n_variables + n_factors + (((n_factors * n_factors) - n_factors) // 2))$$

If *None*, nothing will be bounded. Defaults to *None*.
- **max_iter** (*int*, *optional*) – The maximum number of iterations for the optimization routine. Defaults to 200.
- **tol** (*float* or *None*, *optional*) – The tolerance for convergence. Defaults to *None*.
- **disp** (*bool*, *optional*) – Whether to print the scipy optimization fmin message to standard output. Defaults to *True*.

Raises `ValueError` – If *is_cov_matrix* is *True*, and *n_obs* is not provided.

model

The model specification object.

Type `ModelSpecification`

loadings_

The factor loadings matrix.

Type `numpy array`

error_vars_

The error variance matrix

Type `numpy array`

factor_varcovs_

The factor covariance matrix.

Type `numpy array`

log_likelihood_

The log likelihood from the optimization routine.

Type `float`

aic_

The Akaike information criterion.

Type `float`

bic_

The Bayesian information criterion.

Type `float`

Examples

```

>>> import pandas as pd
>>> from factor_analyzer import (ConfirmatoryFactorAnalyzer,
...                               ModelSpecificationParser)
>>> X = pd.read_csv('tests/data/test11.csv')
>>> model_dict = {"F1": ["V1", "V2", "V3", "V4"],
...               "F2": ["V5", "V6", "V7", "V8"]}
>>> model_spec = ModelSpecificationParser.parse_model_specification_from_dict(X,
↳model_dict)
>>> cfa = ConfirmatoryFactorAnalyzer(model_spec, disp=False)
>>> cfa.fit(X.values)
>>> cfa.loadings_
array([[0.99131285, 0.          ],
       [0.46074919, 0.          ],
       [0.3502267 , 0.          ],
       [0.58331488, 0.          ],
       [0.          , 0.98621042],
       [0.          , 0.73389239],
       [0.          , 0.37602988],
       [0.          , 0.50049507]])
>>> cfa.factor_varcovs_
array([[1.          , 0.17385704],
       [0.17385704, 1.          ]])
>>> cfa.get_standard_errors()
(array([[0.06779949, 0.          ],
       [0.04369956, 0.          ],
       [0.04153113, 0.          ],
       [0.04766645, 0.          ],
       [0.          , 0.06025341],
       [0.          , 0.04913149],
       [0.          , 0.0406604 ],
       [0.          , 0.04351208]]),
 array([0.11929873, 0.05043616, 0.04645803, 0.05803088,
        0.10176889, 0.06607524, 0.04742321, 0.05373646]))
>>> cfa.transform(X.values)
array([[ -0.46852166, -1.08708035],
       [ 2.59025301,  1.20227783],
       [-0.47215977,  2.65697245],
       ...,
       [-1.5930886 , -0.91804114],
       [ 0.19430887,  0.88174818],
       [-0.27863554, -0.7695101 ]])

```

fit (*X*, *y=None*)

Perform confirmatory factor analysis.

Parameters

- **X** (*array-like*) – The data to use for confirmatory factor analysis. If this is just a covariance matrix, make sure *is_cov_matrix* was set to True.
- **y** (*ignored*) –

Raises

- `ValueError` – If the specification is not None or a `ModelSpecification` object
- `AssertionError` – If *is_cov_matrix*=True and the matrix is not square.
- `AssertionError` – If `len(bounds) != len(x0)`

Examples

```

>>> import pandas as pd
>>> from factor_analyzer import (ConfirmatoryFactorAnalyzer,
...                               ModelSpecificationParser)
>>> X = pd.read_csv('tests/data/test11.csv')
>>> model_dict = {"F1": ["V1", "V2", "V3", "V4"],
...               "F2": ["V5", "V6", "V7", "V8"]}
>>> model_spec = ModelSpecificationParser.parse_model_specification_from_
↳dict(X, model_dict)
>>> cfa = ConfirmatoryFactorAnalyzer(model_spec, disp=False)
>>> cfa.fit(X.values)
>>> cfa.loadings_
array([[0.99131285, 0.          ],
       [0.46074919, 0.          ],
       [0.3502267 , 0.          ],
       [0.58331488, 0.          ],
       [0.          , 0.98621042],
       [0.          , 0.73389239],
       [0.          , 0.37602988],
       [0.          , 0.50049507]])

```

get_model_implied_cov()

Get the model-implied covariance matrix (sigma), if the model has been estimated.

Returns model_implied_cov – The model-implied covariance matrix.

Return type numpy array

Examples

```

>>> import pandas as pd
>>> from factor_analyzer import (ConfirmatoryFactorAnalyzer,
...                               ModelSpecificationParser)
>>> X = pd.read_csv('tests/data/test11.csv')
>>> model_dict = {"F1": ["V1", "V2", "V3", "V4"],
...               "F2": ["V5", "V6", "V7", "V8"]}
>>> model_spec = ModelSpecificationParser.parse_model_specification_from_
↳dict(X, model_dict)
>>> cfa = ConfirmatoryFactorAnalyzer(model_spec, disp=False)
>>> cfa.fit(X.values)
>>> cfa.get_model_implied_cov()
array([[2.07938612, 0.45674659, 0.34718423, 0.57824753, 0.16997013,
        0.12648394, 0.06480751, 0.08625868],
       [0.45674659, 1.16703337, 0.16136667, 0.26876186, 0.07899988,
        0.05878807, 0.03012168, 0.0400919 ],
       [0.34718423, 0.16136667, 1.07364855, 0.20429245, 0.06004974,
        0.04468625, 0.02289622, 0.03047483],
       [0.57824753, 0.26876186, 0.20429245, 1.28809317, 0.10001495,
        0.07442652, 0.03813447, 0.05075691],
       [0.16997013, 0.07899988, 0.06004974, 0.10001495, 2.0364391 ,
        0.72377232, 0.37084458, 0.49359346],
       [0.12648394, 0.05878807, 0.04468625, 0.07442652, 0.72377232,
        1.48080077, 0.27596546, 0.36730952],
       [0.06480751, 0.03012168, 0.02289622, 0.03813447, 0.37084458,
        0.27596546, 1.11761918, 0.1882011 ],
       [0.08625868, 0.0400919 , 0.03047483, 0.05075691, 0.49359346,
        0.36730952, 0.1882011 , 1.11761918]])

```

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```
[0.08625868, 0.0400919 , 0.03047483, 0.05075691, 0.49359346,
 0.36730952, 0.1882011 , 1.28888233]])
```

get_standard_errors()

Get the standard errors from the implied covariance matrix and implied means.

Returns

- **loadings_se** (*numpy array*) – The standard errors for the factor loadings.
- **error_vars_se** (*numpy array*) – The standard errors for the error variances.

Examples

```
>>> import pandas as pd
>>> from factor_analyzer import (ConfirmatoryFactorAnalyzer,
...                               ModelSpecificationParser)
>>> X = pd.read_csv('tests/data/test11.csv')
>>> model_dict = {"F1": ["V1", "V2", "V3", "V4"],
...               "F2": ["V5", "V6", "V7", "V8"]}
>>> model_spec = ModelSpecificationParser.parse_model_specification_from_
↳dict(X, model_dict)
>>> cfa = ConfirmatoryFactorAnalyzer(model_spec, disp=False)
>>> cfa.fit(X.values)
>>> cfa.get_standard_errors()
(array([[0.06779949, 0.          ],
        [0.04369956, 0.          ],
        [0.04153113, 0.          ],
        [0.04766645, 0.          ],
        [0.          , 0.06025341],
        [0.          , 0.04913149],
        [0.          , 0.0406604 ],
        [0.          , 0.04351208]]),
array([0.11929873, 0.05043616, 0.04645803, 0.05803088,
        0.10176889, 0.06607524, 0.04742321, 0.05373646]))
```

transform(X)

Get the factor scores for new data set.

Parameters X (*array-like, shape (n_samples, n_features)*) – The data to score using the fitted factor model.

Returns scores – The latent variables of X.

Return type *numpy array, shape (n_samples, n_components)*

Examples

```
>>> import pandas as pd
>>> from factor_analyzer import (ConfirmatoryFactorAnalyzer,
...                               ModelSpecificationParser)
>>> X = pd.read_csv('tests/data/test11.csv')
>>> model_dict = {"F1": ["V1", "V2", "V3", "V4"],
...               "F2": ["V5", "V6", "V7", "V8"]}
>>> model_spec = ModelSpecificationParser.parse_model_specification_from_
↳dict(X, model_dict)
```

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```

>>> cfa = ConfirmatoryFactorAnalyzer(model_spec, disp=False)
>>> cfa.fit(X.values)
>>> cfa.transform(X.values)
array([[ -0.46852166, -1.08708035],
       [ 2.59025301,  1.20227783],
       [-0.47215977,  2.65697245],
       ...,
       [-1.5930886 , -0.91804114],
       [ 0.19430887,  0.88174818],
       [-0.27863554, -0.7695101 ]])

```

References

<https://www.ncbi.nlm.nih.gov/pmc/articles/PMC6157408/>

class `factor_analyzer.confirmatory_factor_analyzer.ModelSpecification` (*loadings, n_factors, n_variables, factor_names=None, variable_names=None*)

Bases: `object`

A class to encapsulate the model specification for CFA. This class contains a number of specification properties that are used in the CFA procedure.

Parameters

- **loadings** (*array-like*) – The factor loadings specification.
- **error_vars** (*array-like*) – The error variance specification
- **factor_covs** (*array-like*) – The factor covariance specification.
- **factor_names** (*list of str or None*) – A list of factor names, if available. Defaults to None.
- **variable_names** (*list of str or None*) – A list of variable names, if available. Defaults to None.

loadings

The factor loadings specification.

Type numpy array

error_vars

The error variance specification

Type numpy array

factor_covs

The factor covariance specification.

Type numpy array

n_factors

The number of factors.

Type int

n_variables

The number of variables.

Type int

n_lower_diag

The number of elements in the *factor_covs* array, which is equal to the lower diagonal of the factor covariance matrix.

Type int

loadings_free

The indexes of “free” factor loading parameters.

Type numpy array

error_vars_free

The indexes of “free” error variance parameters.

Type numpy array

factor_covs_free

The indexes of “free” factor covariance parameters.

Type numpy array

factor_names

A list of factor names, if available.

Type list of str or None

variable_names

A list of variable names, if available.

Type list of str or None

copy ()**error_vars****error_vars_free****factor_covs****factor_covs_free****factor_names****get_model_specification_as_dict ()**

Get the model specification as a dictionary.

Returns **model_specification** – The model specification keys and values, as a dictionary.

Return type dict

loadings**loadings_free****n_factors****n_lower_diag****n_variables****variable_names**

class factor_analyzer.confirmatory_factor_analyzer.**ModelSpecificationParser**

Bases: `object`

A class to generate the model specification for CFA. This class includes two static methods to generate the `ModelSpecification` object from either a dictionary or a numpy array.

static parse_model_specification_from_array (*X*, *specification=None*)

Generate the model specification from an array. The columns should correspond to the factors, and the rows should correspond to the variables. If this method is used to create the `ModelSpecification`, then no factor names and variable names will be added as properties to that object.

Parameters

- **x** (*array-like*) – The data set that will be used for CFA.
- **specification** (*array-like or None*) – An array with the loading details. If `None`, the matrix will be created assuming all variables load on all factors. Defaults to `None`.

Returns A model specification object

Return type `ModelSpecification`

Raises `ValueError` – If *specification* is not in the expected format.

Examples

```
>>> import pandas as pd
>>> import numpy as np
>>> from factor_analyzer import (ConfirmatoryFactorAnalyzer,
...                               ModelSpecificationParser)
>>> X = pd.read_csv('tests/data/test11.csv')
>>> model_array = np.array([[1, 1, 1, 1, 0, 0, 0, 0], [0, 0, 0, 0, 1, 1, 1,
↵1]])
>>> model_spec = ModelSpecificationParser.parse_model_specification_from_
↵array(X,
...
↵ model_array)
```

static parse_model_specification_from_dict (*X*, *specification=None*)

Generate the model specification from a dictionary. The keys in the dictionary should be the factor names, and the values should be the feature names. If this method is used to create the `ModelSpecification`, then factor names and variable names will be added as properties to that object.

Parameters

- **x** (*array-like*) – The data set that will be used for CFA.
- **specification** (*dict or None*) – A dictionary with the loading details. If `None`, the matrix will be created assuming all variables load on all factors. Defaults to `None`.

Returns A model specification object

Return type `ModelSpecification`

Raises `ValueError` – If *specification* is not in the expected format.

Examples

```
>>> import pandas as pd
>>> from factor_analyzer import (ConfirmatoryFactorAnalyzer,
...                               ModelSpecificationParser)
>>> X = pd.read_csv('tests/data/test11.csv')
>>> model_dict = {"F1": ["V1", "V2", "V3", "V4"],
...               "F2": ["V5", "V6", "V7", "V8"]}
>>> model_spec = ModelSpecificationParser.parse_model_specification_from_
↳dict(X, model_dict)
```

3.1.4 factor_analyzer.rotator Module

Class to perform various rotations of factor loading matrices.

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author Nitin Madnani (nmadnani@ets.org)

organization Educational Testing Service

date 2021-10-18

class `factor_analyzer.rotator.Rotator` (*method='varimax', normalize=True, power=4, kappa=0, gamma=0, delta=0.01, max_iter=500, tol=1e-05*)

Bases: `sklearn.base.BaseEstimator`

The Rotator class takes an (unrotated) factor loading matrix and performs one of several rotations.

Parameters

- **method** (*str, optional*) –

The factor rotation method. Options include:

- varimax (orthogonal rotation)
- promax (oblique rotation)
- oblimin (oblique rotation)
- oblimax (orthogonal rotation)
- quartimin (oblique rotation)
- quartimax (orthogonal rotation)
- equamax (orthogonal rotation)
- geomin_obl (oblique rotation)
- geomin_ort (orthogonal rotation)

Defaults to 'varimax'.

- **normalize** (*bool or None, optional*) – Whether to perform Kaiser normalization and de-normalization prior to and following rotation. Used for varimax and promax rotations. If None, default for promax is False, and default for varimax is True. Defaults to None.
- **power** (*int, optional*) – The power to which to raise the promax loadings (minus 1). Numbers should generally range from 2 to 4. Defaults to 4.

- **kappa** (*int, optional*) – The kappa value for the equamax objective. Ignored if the method is not ‘equamax’. Defaults to 0.
- **gamma** (*int, optional*) – The gamma level for the oblimin objective. Ignored if the method is not ‘oblimin’. Defaults to 0.
- **delta** (*float, optional*) –
The delta level for geomin objectives. Ignored if the method is not ‘geomin_*’. Defaults to 0.01.
- **max_iter** (*int, optional*) – The maximum number of iterations. Used for varimax and oblique rotations. Defaults to 1000.
- **tol** (*float, optional*) – The convergence threshold. Used for varimax and oblique rotations. Defaults to $1e-5$.

loadings_

The loadings matrix

Type numpy array, shape (n_features, n_factors)

rotation_

The rotation matrix

Type numpy array, shape (n_factors, n_factors)

psi_

The factor correlations matrix. This only exists if the rotation is oblique.

Type numpy array or `None`

Notes

Most of the rotations in this class are ported from R’s *GPARotation* package.

References

[1] <https://cran.r-project.org/web/packages/GPARotation/index.html>

Examples

```
>>> import pandas as pd
>>> from factor_analyzer import FactorAnalyzer, Rotator
>>> df_features = pd.read_csv('test02.csv')
>>> fa = FactorAnalyzer(rotation=None)
>>> fa.fit(df_features)
>>> rotator = Rotator()
>>> rotator.fit_transform(fa.loadings_)
array([[ -0.07693215,  0.04499572,  0.76211208],
       [ 0.01842035,  0.05757874,  0.01297908],
       [ 0.06067925,  0.70692662, -0.03311798],
       [ 0.11314343,  0.84525117, -0.03407129],
       [ 0.15307233,  0.5553474 , -0.00121802],
       [ 0.77450832,  0.1474666 ,  0.20118338],
       [ 0.7063001 ,  0.17229555, -0.30093981],
       [ 0.83990851,  0.15058874, -0.06182469],
```

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```
[ 0.76620579,  0.1045194 , -0.22649615],
 [ 0.81372945,  0.20915845,  0.07479506]])
```

fit (*X*, *y=None*)

Computes the factor rotation.

Parameters

- **X** (*array-like*) – The factor loading matrix (*n_features*, *n_factors*)
- **y** (*Ignored*) –

Returns**Return type** *self***Example**

```
>>> import pandas as pd
>>> from factor_analyzer import FactorAnalyzer, Rotator
>>> df_features = pd.read_csv('test02.csv')
>>> fa = FactorAnalyzer(rotation=None)
>>> fa.fit(df_features)
>>> rotator = Rotator()
>>> rotator.fit(fa.loadings_)
```

fit_transform (*X*, *y=None*)

Computes the factor rotation, and returns the new loading matrix.

Parameters

- **X** (*array-like*) – The factor loading matrix (*n_features*, *n_factors*)
- **y** (*Ignored*) –

Returns *loadings_* – The loadings matrix (*n_features*, *n_factors*)**Return type** *numpy array*, shape (*n_features*, *n_factors*)**Raises** *ValueError* – If the *method* is not in the list of acceptable methods.**Example**

```
>>> import pandas as pd
>>> from factor_analyzer import FactorAnalyzer, Rotator
>>> df_features = pd.read_csv('test02.csv')
>>> fa = FactorAnalyzer(rotation=None)
>>> fa.fit(df_features)
>>> rotator = Rotator()
>>> rotator.fit_transform(fa.loadings_)
array([[ -0.07693215,  0.04499572,  0.76211208],
       [ 0.01842035,  0.05757874,  0.01297908],
       [ 0.06067925,  0.70692662, -0.03311798],
       [ 0.11314343,  0.84525117, -0.03407129],
       [ 0.15307233,  0.5553474 , -0.00121802],
       [ 0.77450832,  0.1474666 ,  0.20118338],
       [ 0.7063001 ,  0.17229555, -0.30093981],
```

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```
[ 0.83990851, 0.15058874, -0.06182469],  
[ 0.76620579, 0.1045194 , -0.22649615],  
[ 0.81372945, 0.20915845, 0.07479506]])
```

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